An Analysis of the Single Sensor Bearings-Only Tracking Problem†

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***Abstract*— In this paper we examine the single sensor, bearings-only target tracking problem. This problem is known to be difficult due to the potential unobservability of elements of the target's state and the high degree of nonlinearity in the measurement process. We compare the performance of three different tracking algorithms. The algorithms are of varying degrees of computational complexity. The results of these comparisons can be used to gain insight into the degree of difficulty caused by each of these two issues. The understanding this provides can aid in the selection of an appropriate target tracking filter for a given application.**

Keywords: bearings-only tracking, nonlinear filtering, log polar coordinates

I. Introduction

The bearings-only tracking problem is a common one in radar and sonar environments. However, it is known to be a difficult problem, particularly when there is only a single sensor. There are two main issues which contribute to making this a hard problem. The first is the high degree of nonlinearity of the measurement process [1]. The second is that the target state is not fully observable unless the sensor platform "out manoeuvres" the target [2]. If this does not happen, the sensor does not have sufficient information to estimate the range to the target.

One of the more common methods for dealing with a nonlinear model is to use the extended Kalman filter (EKF) [3]. However, this is known to lack robustness and can diverge if the degree of nonlinearity of the system is high or if the filter is poorly initialised. More sophisticated approaches include the unscented Kalman filter (UKF) [4] and the particle filter [5]. A particle filter based tracker for the bearing-only problem was compared with other common approaches in [6].

All of the approaches mentioned use a single filter to estimate the target's state. An alternative method is to model the nonlinearity by a sum of Gaussians [7]. The Gaussian sum approximation approach has been used for the single sensor bearings-only problem to construct a multihypothesis Kalman filter [8]; a range-parameterised EKF tracker [9]; and a Gaussian sum measurement presentation [10].

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A common tactic for dealing with the possibile unobservability of the target's range is to use the modified polar coordinate (MPC) basis of [11]. In this coordinate basis, the target's dynamical model is nonlinear but the measurement model is linear. More importantly, the unobservable and observable components of the target's state vector are decoupled. Use of this coordinate basis improves the stability and robustness of an EKF-based tracking filter. Recently, a variant of the MPC system was proposed in [12] called the log polar coordinate (LPC) basis. As pointed out in [12] an advantage of LPC over MPC is that it is possible to derive a closed-form expression for the Cramér-Rao lower bound. In addition, preliminary results suggest that a further advantage of LPC is that an EKF using LPC is more robust than one that makes use of MPC [1].

In this paper, we examine the effects of *nonlinearity* and *unobservability* on the degree of difficulty of the single-sensor bearings-only tracking problem. We do this by comparing three filtering algorithms for this problem. These are

1) a Gaussian sum measurement approximation filter;
2) an extended Kalman filter using LPC; and
3) a range-parameterised EKF using LPC.

The first filter focuses on dealing with the issues arising from the nonlinearities in the problem, while the second concentrates on the unobservability issue. The third filter seeks to address both problems. By comparing the performance of these three trackers we can determine the contribution of each issue to the overall difficulty of this problem. This can then provide guidelines to determining where to focus resources such as computational effort in applications where resources are constrained.

The next section defines the single-sensor bearings-only tracking problem mathematically. Sections III–V describe the three tracking algorithms. In Section VI the problem of initialising these filters is discussed. This is an important point as poor initialisation generally leads to track loss, even for the most robust filter. Finally, in Section VII the algorithms are compared using a simple simulation scenario.

II. Single-Sensor Bearings-Only Tracking Problem

In this paper, we consider only 2D tracking problems and we define the x coordinate to be East and the y coordinate as North. The bearing, β, is measured clockwise from North. We use boldface to indicate vectors with scalars in normal font.

The position of the target in 2D is given by (x^t, y^t), and its speed and acceleration given by $(\dot{x}^t, \dot{y}^t)$ and $(\ddot{x}^t, \ddot{y}^t)$ respectively, where the superscript t is used to indicate these are target state variables. The corresponding values for the sensor platform, or *observer*, are (x^o, y^o), $(\dot{x}^o, \dot{y}^o)$ and $(\ddot{x}^o, \ddot{y}^o)$, as indicated by the superscript o. Assuming constant velocity motion [2] for the target, the relative target state vector at time t_k is

$$\mathbf{x}_k \triangleq \begin{bmatrix} (x_k^t - x_k^o) & (y_k^t - y_k^o) & (\dot{x}_k^t - \dot{x}_k^o) & (\dot{y}_k^t - \dot{y}_k^o) \end{bmatrix}' \tag{1}$$

and the sensor measurement is

$$z_k = \beta_k + v_k \tag{2}$$

where $v_k \sim \mathcal{N}(0, \sigma_\beta^2)$ and

$$\beta_k = \arctan\left(\frac{x_k}{y_k}\right) \tag{3}$$

The target state dynamical equation in Cartesian coordinates is

$$\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{4}$$

where

$$\mathbf{A}_k \triangleq \begin{bmatrix} 1 & 0 & T_k & 0 \\ 0 & 1 & 0 & T_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

and $T_k = t_k - t_{k-1}$ and the elements of the perturbation term, $\mathbf{w}_{k-1}$, are given by

$$w_1 = \int_{t_{k-1}}^{t_k} (t_k - u)(\ddot{x}_u^t - \ddot{x}_u^o)du \tag{6}$$

$$w_2 = \int_{t_{k-1}}^{t_k} (t_k - u)(\ddot{y}_u^t - \ddot{y}_u^o)du \tag{7}$$

$$w_3 = \int_{t_{k-1}}^{t_k} (\ddot{x}_u^t - \ddot{x}_u^o)du \tag{8}$$

$$w_4 = \int_{t_{k-1}}^{t_k} (\ddot{y}_u^t - \ddot{y}_u^o)du \tag{9}$$

(10)

where w_i is the i-th element of the vector $\mathbf{w}$.

III. Gaussian Sum Measurement Approximation Filter

When tracking using bearings-only or range and bearing measurements, the uncertainty region in Cartesian coordinates is wedge shaped [8], [9]. Approximating this uncertainty region with a single Gaussian can lead to poor performance. The tracking algorithm described in this section is based on those in [13] and [10]. In these works, the measurement is approximated by a sum of Gaussians as first proposed in [7]. In contrast to the range-parameterised approach first described by Peach [9], only a single equation is used for the state dynamics. Instead, only the measurement equation that is approximated by a sum of Gaussians. These give rise to a weighted sum of possible innovations, which are combined to produce a single state estimate. Figure 1 shows the measurement uncertainty region in Cartesian coordinates given a bearing measurement of $50°$ with an measurement standard deviation of $\sigma_\beta = 1°$. All that is known about the true target range is that it lies in the interval $[2, 20]$ km. This region is approximated using $N_G = 10$ Gaussians using the method described below. Figure 2 shows the accuracy of the sum of Gaussians approximation to the uniform uncertainty in the range measurement.

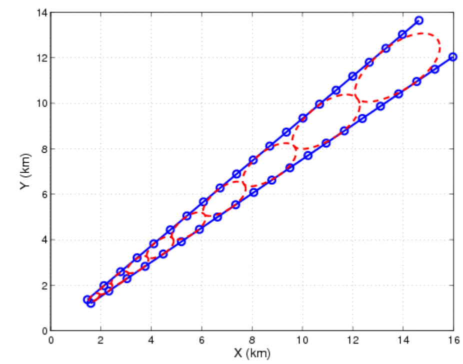

Fig. 1. Measurement uncertainty region in Cartesian coordinates approximated by 10 Gaussians.

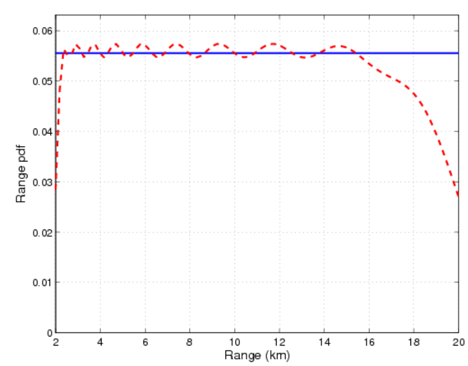

Fig. 2. Accuracy of the sum of Gaussians approximation to uniform uncertainty in range.

More precisely, the Gaussian sum measurement approximation bearings-only tracking algorithm operates as follows. The target state is given in Cartesian coordinates. At the start of scan k, the input to the algorithm is the filtered state estimate, $\hat{\mathbf{x}}_{k-1|k-1}$ and its associated error covariance matrix, $\mathbf{P}_{k-1|k-1}$ from the previous iteration. The *prediction step* is then the standard Kalman filter prediction step using the dynamic equation (4).

Then, assuming the true target state lies in the interval

$$\left[\begin{array}{c} (\hat{x} - 3C\sqrt{\mathbf{P}(1,1)}, \hat{x} + 3C\sqrt{\mathbf{P}(1,1)}) \\ (\hat{y} - 3C\sqrt{\mathbf{P}(2,2)}, \hat{y} + 3C\sqrt{\mathbf{P}(2,2)}) \end{array} \right] \tag{11}$$

where $\mathbf{P}(i,j)$ is the (i,j)-th element of the matrix $\mathbf{P}$ and C is a user-specified parameter, compute the minimum and maximum range to the target, (r_{min}, r_{max}). This interval is then divided into N_G Gaussians using the method in [8], [9]. That is, let the centre, r_c^i, and length, r_d^i, of the i-th interval be given by

$$\frac{r_d^i}{r_c^i} = c \tag{12}$$

where

$$c = \frac{2(\eta - 1)}{\eta + 1}, \qquad \eta = \left(\frac{r_{max}}{r_{min}}\right)^{\frac{1}{N_G}} \tag{13}$$

This non-uniform division of the range interval insures the coefficient of variation is the same in each interval. This is believe to assist in the tracking performance [9]. For each range subinterval, define the weighting probabilities as

$$\gamma_k^i = \frac{r_d^i}{\sum_i r_d^i} \tag{14}$$

For each range subinterval, compute a pseudo-measurement in Cartesian coordinates using the centre of the interval as the range estimate and the unbiased converted measurement method of [14]. That is, assuming the true measurement noise is Gaussian, the pseudo-measurement is

$$\mathbf{z}_k^i \triangleq \left[\begin{array}{cc} \lambda_\beta^{-1} r_c^i \sin z_k & \lambda_\beta^{-1} r_c^i \cos z_k \end{array} \right]' \tag{15}$$

where $\lambda_\beta = \exp(-0.5\sigma_\beta^2)$ and σ_β is the measurement noise standard deviation. The corresponding measurement noise matrix for the i-interval, $\mathbf{R}^i$, is given by the equations

$$\begin{aligned} \mathbf{R}^i(1,1) &= (\lambda_\beta^{-2} - 2)(r_c^i)^2 \sin^2 z_k \\ &\quad + \tfrac{1}{2}((r_c^i)^2 + \sigma_r^2)(1 - \lambda_\beta^4 \cos 2z_k) && (16) \\ \mathbf{R}^i(2,2) &= (\lambda_\beta^{-2} - 2)(r_c^i)^2 \cos^2 z_k \\ &\quad + \tfrac{1}{2}((r_c^i)^2 + \sigma_r^2)(1 + \lambda_\beta^4 \cos 2z_k) && (17) \\ \mathbf{R}^i(1,2) &= (\lambda_\beta^{-2} - 2)(r_c^i)^2 \cos z_k \sin z_k \\ &\quad + \tfrac{1}{2}((r_c^i)^2 + \sigma_r^2)\lambda_\beta^4 \sin 2z_k && (18) \end{aligned}$$

where $\sigma_r = \frac{r_d^i}{6}$. Then, using each pseudo-measurement, do a standard Kalman filter update step to produce a filtered state estimate, $\hat{\mathbf{x}}_{k|k}^i$, and its associated error covariance matrix, $\mathbf{P}_{k|k}^i$, for each subinterval. Finally, compute the weighted average of the estimate from each subinterval, i.e.

$$\hat{\mathbf{x}}_{k|k} = \sum_{i=1}^{N_G} \gamma_k^i \hat{\mathbf{x}}_{k|k}^i \tag{19}$$

$$\mathbf{P}_{k|k} = \sum_{i=1}^{N_G} \gamma_k^i \left\{ \mathbf{P}_{k|k}^i + (\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k})(\hat{\mathbf{x}}_{k|k}^i - \hat{\mathbf{x}}_{k|k})' \right\} \tag{20}$$

Note, it is not necessary to combine all the Gaussians into a single Gaussian approximation at the end of each scan. More sophisticated approaches are possible. However, they lead to an exponentially increasing number of Gaussians with each scan, unless mixture reduction algorithms such as those described in [15] are used.

IV. LPC-EKF TRACKER

The equations for the extended Kalman filter are well known and can be found in references such [3]. For reasons of space, they are not repeated here. The relative state vector in LPC is given by

$$\mathbf{y}_k \triangleq \left[\begin{array}{cccc} \dot{\beta}_k & \dot{\rho}_k & \beta_k & \rho_k \end{array} \right]' \tag{21}$$

where

$$r_k \triangleq \sqrt{x_k^2 + y_k^2} \tag{22}$$

$$\rho_k \triangleq \ln(r_k) \tag{23}$$

$$\dot{r}_k \triangleq \frac{x_k \dot{x}_k + y_k \dot{y}_k}{r_k} \tag{24}$$

$$\dot{\beta}_k \triangleq \frac{\dot{x}_k y_k - x_k \dot{y}_k}{r_k^2} \tag{25}$$

and hence $\dot{\rho}_k = \frac{\dot{r}_k}{r_k}$. As with MPC, the first three elements of the LPC state are always observable. The final element is unobservable unless the observer manoeuvres appropriately. However, unlike MPC, this coordinate basis ensures that the estimated range is always positive.

It is straightforward to show that the conversions between Cartesian and LPC and vice versa are given by (dropping the time index for clarity)

$$\mathbf{x} = \exp(y_4) \left[\begin{array}{c} \sin y_3 \\ \cos y_3 \\ y_2 \sin y_3 + y_1 \cos y_3 \\ y_2 \cos y_3 - y_1 \sin y_3 \end{array} \right] \tag{26}$$

and

$$\mathbf{y} = \left[\begin{array}{c} \frac{x_2 x_3 - x_1 x_4}{x_1^2 + x_2^2} \\ \frac{x_1 x_3 + x_2 x_4}{x_1^2 + x_2^2} \\ \arctan\left(\frac{x_1}{x_2}\right) \\ \frac{1}{2}\ln\left(x_1^2 + x_2^2\right) \end{array} \right] \tag{27}$$

Using an approach similar to that of [11] it can be shown that the target dynamics in LPC are given by

$$y_1(t_k) = \frac{S_1 S_4 - S_2 S_3}{S_3^2 + S_4^2} \tag{28}$$

$$y_2(t_k) = \frac{S_1 S_3 + S_2 S_4}{S_3^2 + S_4^2} \tag{29}$$

$$y_3(t_k) = y_3(t_{k-1}) + \arctan\left(\frac{S_3}{S_4}\right) \tag{30}$$

$$y_4(t_k) = y_4(t_{k-1}) + \tfrac{1}{2}\ln\left(S_3^2 + S_4^2\right) \tag{31}$$

where

$$S_1 = y_1 + \exp(-y_4)\left[w_3 \cos(y_3) - w_4 \sin(y_3)\right] \tag{32}$$

$$S_2 = y_2 + \exp(-y_4)\left[w_3 \sin(y_3) + w_4 \cos(y_3)\right] \tag{33}$$

$$S_3 = T y_1 + \exp(-y_4)\left[w_1 \cos(y_3) - w_2 \sin(y_3)\right] \tag{34}$$

$$S_4 = 1 + T y_2 + \exp(-y_4)\left[w_1 \sin(y_3) + w_2 \cos(y_3)\right] \tag{35}$$

and the values on the right-hand side of (32)–(35) are elements of $\mathbf{y}_{k-1}$ and $\mathbf{w}_{k-1}$. Given (28)–(31) it is straightforward, if tedious, to compute the Jacobian matrix for the EKF prediction step. The interested reader is referred to [1] for details.

V. Range-Parameterised LPC-EKF

This filter, hereafter called the RP-LPC-EKF, is based on the range-parameterised EKF of Peach [9]. This filter is initialised with an user-specified minimum and maximum range to the target, r_{min} and r_{max}. This range interval is then divided into N_G subinterval using the method described in Section III. Then, an EKF is initialised for each range subinterval using the method described in Section VI.

For each individual subinterval, the target state is estimated using the extended Kalman filter algorithm. Assuming the measurement noise is Gaussian, the marginal measurement likelihood from each EKF, $p_z^i = p(z_k|i)$, can be easily computed. The weights are then updated using

$$\gamma_k^i = \frac{p_z^i \gamma_{k-1}^i}{\sum_{i=1}^{N_G} p_z^i \gamma_{k-1}^i} \tag{36}$$

For output only, the average state estimate in LPC and its associated error covariance is given by

$$\hat{\mathbf{y}}_{k|k} = \sum_{i=1}^{N_G} \gamma_k^i \hat{\mathbf{y}}_{k|k}^i \tag{37}$$

$$\mathbf{P}_{k|k} = \sum_{i=1}^{N_G} \gamma_k^i \left\{ \mathbf{P}_{k|k}^i + (\hat{\mathbf{y}}_{k|k}^i - \hat{\mathbf{y}}_{k|k})(\hat{\mathbf{y}}_{k|k}^i - \hat{\mathbf{y}}_{k|k})' \right\} \tag{38}$$

where $\hat{\mathbf{y}}_{k|k}^i$ and $\mathbf{P}_{k|k}^i$ are the estimated state and its associated error covariance from the i-th filter. Note, in contrast to the Gaussian sum measurement approximation filter, an individual filter is maintained for each range subinterval.

VI. Initialisation

Accurate initialisation is crucial to obtaining effective tracking performance for the single sensor bearings-only problem. The method used in this paper is the one described in Section 6.4.1 of [5]. A more sophisticated method was first trialled, making using of the unbiased conversion of [14] but this proved to be less robust than the method described here. This is probably due to the relatively low measurement noise used in the scenario considered here. In such cases, the bias conversion factor, λ_β, is close to unity and the resulting error covariance matrix is ill-conditioned.

Each filter is initialised using the first bearing measurement z_1 and the known measurement noise standard deviation σ_β. An initial range estimate $\bar{r}$ and associated range measurement standard deviation σ_r are then used to compute an initial estimate of the target position. That is, the position elements of $\hat{\mathbf{x}}_{1|1}$ are given by

$$\hat{\mathbf{x}}(1) = \bar{r} \sin z_1 \tag{39}$$

$$\hat{\mathbf{x}}(2) = \bar{r} \cos z_1 \tag{40}$$

The corresponding error covariance terms are

$$\mathbf{P}(1,1) = \sigma_r^2 \sin^2 z_1 + \bar{r}^2 \sigma_\beta^2 \cos^2 z_1 \tag{41}$$

$$\mathbf{P}(2,2) = \sigma_r^2 \cos^2 z_1 + \bar{r}^2 \sigma_\beta^2 \sin^2 z_1 \tag{42}$$

$$\mathbf{P}(1,2) = (\sigma_r^2 - \bar{r}^2 \sigma_\beta^2) \sin z_1 \cos z_1 \tag{43}$$

For the Gaussian sum measurement approximation filter and the RP-LPC-EKF the values of $\bar{r}$ and σ_r are given by the mean and length of each range subinterval as described in Section III. For the LPC-EKF these values are user-specified parameters.

To set the velocity components we require estimates of the initial heading and speed of the target. The initial course of the target is estimated as $\bar{c} = z_1 + \pi$. The minimum and maximum speed of the target, s_{min} and s_{max}, are user-specified parameters. The speed interval is divided using the same method as for the range interval, with the lower speed intervals associated with the closer range intervals as in [5], i.e. the i-th filter is initialised with mean values for the range and speed of (r_c^i, s_c^i).

The uncertainty in the target's course is assumed to be uniform in the range $[\frac{-\pi}{2}, \frac{\pi}{2}]$, thus it has a standard deviation of $\sigma_c = \frac{\pi}{\sqrt{12}}$. With these assumptions, the velocity components of the state vector are then initialised using

$$\hat{\mathbf{x}}(3) = \bar{s} \sin \bar{c} - \dot{x}_1^o \tag{44}$$

$$\hat{\mathbf{x}}(4) = \bar{s} \cos \bar{c} - \dot{y}_1^o \tag{45}$$

The velocity covariance terms are given by

$$\mathbf{P}(3,3) = \sigma_s^2 \sin^2 \bar{c} + \bar{s}^2 \sigma_c^2 \cos^2 \bar{c} \tag{46}$$

$$\mathbf{P}(4,4) = \sigma_s^2 \cos^2 \bar{c} + \bar{s}^2 \sigma_c^2 \sin^2 \bar{c} \tag{47}$$

$$\mathbf{P}(3,4) = (\sigma_s^2 - \bar{s}^2 \sigma_c^2) \sin \bar{c} \cos \bar{c} \tag{48}$$

For the Gaussian sum measurement approximation filter and the RP-LPC-EKF the values of $\bar{s}$ and σ_s are given by the mean and length of each speed subinterval as described in Section III. For the LPC-EKF these values are user-specified parameters.

Given the initial estimate and its associated error covariance in Cartesian coordinates, the LPC-based filters are initialised by noting that we can write (27) as $\mathbf{y} = \mathbf{f}(\mathbf{x})$. Then, using a first-order approximation, the initial estimate in LPC is

$$\hat{\mathbf{y}}_{1|1} = \mathbf{f}(\hat{\mathbf{x}}_{1|1}) \tag{49}$$

and its associated error covariance is given by

$$\mathbf{P}_{1|1}^{LPC} = \mathbf{F} \mathbf{P}_{1|1} \mathbf{F}' \tag{50}$$

where $\mathbf{F}$ is

$$\mathbf{F} = \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_{1|1}} \tag{51}$$

VII. Simulations

The three tracking algorithms where compared using 1000 Monte Carlo simulations of a simple tracking scenario. This scenario is based on the one used in Section 6.5.1 in [5] for a non-manoeuvring target. Both the observer and the target are submarines, with the observer initially at the origin of the coordinate frame. The initial range and bearing to the target are 5km and 80° respectively. The observer's trajectory has three distinct segments – constant velocity motion for 13 minutes; a coordinated turn through 120° for 4 minutes; followed by constant velocity motion for a further 13 minutes. The observer moves with a constant speed of 5 knots while the target's speed is 4 knots. The target follows constant velocity motion with a heading of -140°. The bearing measurement noise standard deviation was 1° and the sampling interval was 1 minute.

The range and speed parameters used for initialisation are shown in Table I. For the range-parameterised EKF and the Gaussian sum measurement filter the range interval was divided into $N_G = 5$ subintervals. The covariance scaling factor, for the Gaussian sum measurement filter was set to $C = 10$. For the LPC-EKF filter, the range and speed standard deviations were set to $\sigma_r = 2$ km and $\sigma_s = 2$ kts respectively. Even though the true target dynamics were deterministic, a minor amount of process noise was added to the LPC-based filters. This ensured that the gain in the filter remained sufficiently high.

TABLE I

RANGE AND SPEED INITIALISATION PARAMETERS

	Minimum	Maximum	Mean
Range	$r_{min} = 1km$	$r_{max} = 25km$	$\bar{r} = 13km$
Speed	$s_{min} = 2kts$	$s_{max} = 15kts$	$\bar{s} = 8.5kts$

Once again, following the approach in [5], we evaluated the performance of the tracking algorithms using the following three metrics

1) root-mean square (RMS) position error at the final sampling time;
2) root time-averaged mean square (RTAMS) error; and
3) the number of divergent tracks.

Using the metrics of [5], a track was deemed to have *diverged* if the estimated position error at any time was greater than 20km. The RMS and RTAMS metrics were only computed for non-divergent tracks. The RMS metric at scan k is given by

$$\mathrm{RMS}_k = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(\hat{x}_k^i + x_k^o - x_k^t)^2 + (\hat{y}_k^i + y_k^o - y_k^t)^2} \tag{52}$$

where M is the number of Monte Carlo simulations. The RTAMS metric is given by

$$\mathrm{RTAMS} = \tag{53}$$

$$\sqrt{\frac{1}{M(k_N - k_t)}\sum_{i=1}^{M}\sum_{k=k_t+1}^{k_N}(\hat{x}_k^i + x_k^o - x_k^t)^2 + (\hat{y}_k^i + y_k^o - y_k^t)^2}$$

where k_N is the index of the final scan and k_t is the scan at which the observer completed its turn. The target trajectory is only completely observable once the observer has commenced a manoeuvre, so the RTAMS metric is a measure of how quickly each tracker can refine its track estimates once it has full information on the target.

TABLE II

TRACKER PERFORMANCE METRICS

Algorithm	Final RMS km	RTAMS km	Divergent Tracks
Gaussian Sum	0.32	1.43	0
LPC-EKF	0.24	0.29	0
RP-LPC-EKF	0.26	0.22	1

From the results in Table II we see that all three filters provide similar tracking accuracy by the final scan. The significant differences between the filters are in the speed with which they are able to make use of the range information provided once the observer begins to manoeuvre. The Gaussian sum approximation filter suffers from a relatively slow convergence rate, while the two LPC-based filters show similar behaviour.

The similarity in performance between the LPC-EKF and its range-parameterised equivalent is in contrast to the results of Peach [9]. This may be due, in part, to the particular geometry of this scenario. In [9], the most significant differences between the MPC-EKF and the range-parameterised MPC-EKF occurred when the relative range was small ($<$ 2 km) or large ($>$ 20 km). Another cause is suggested by the results of [1]. These results suggest that by using LPC in place of MPC, the degree of nonlinearity of the problem is reduced, increasing the robustness of an EKF-based filter.

Figures 3–5 show the tracker output from each of the three filters from a single Monte Carlo run. These figures clearly illustrate the relative differences between the filters in respect of accurate performance at the final scan and accurate performance after the observer's manoeuvre.

Using the simplest algorithm, LPC-EKF, as a baseline the runtimes of the other two algorithms were approximately 5 times as long for the Gaussian sum measurement approximation filter and 7 times as long for the range-parameterised LPC-EKF algorithm.

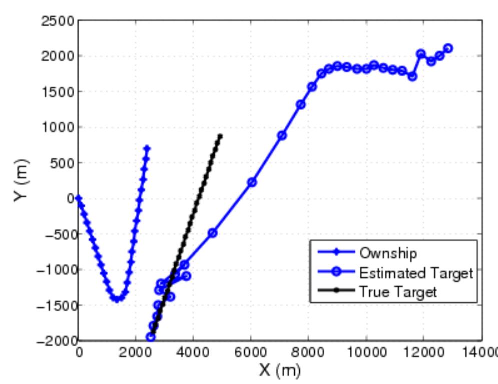

Fig. 3. Tracker output from a single run of the Gaussian sum measurement approximation filter.

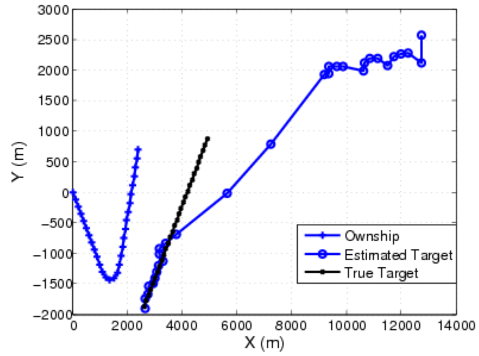

Fig. 4. Tracker output from a single run of the LPC-EKF filter.

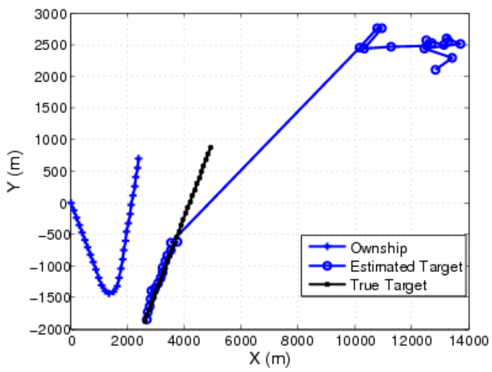

Fig. 5. Tracker output from a single run of the range-parameterised LPC-EKF filter.

VIII. Conclusions

The results of this paper suggest that the key issue in making the single sensor bearings-only tracking problem difficult is the lack of full observability of the target state. By selecting a coordinate basis, such as MPC or LPC, the effect of this issue is greatly ameliorated. It is then possible to use a relatively unsophisticated nonlinear filter and still obtain adequate performance.

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